12^{TH} MATHEMATICS

CHAP.1 APPLICATIONS OF MATRICES AND DETERMINANTS

UNIT:1 Adjoint , Inverse and Orthogonal of a Matrix

Important Results:

i) Adjoint of A: $adjA=\left[A_{ij}\right]^{T}$

ii) Inverse of A: $A^{-1}=\frac{1}{|A|}adj(A)$

iii) Orthogonal Matrix: $AA^{T}=I$ or $A^{-1}=A^{T}$

iv) Properties of Adjoint:

$$A(adjA)=(adjA)A=|A|I$$

$$adj(AB)=(adjB)(adjA)$$

v) Properties of Inverse:

$$(AB)^{-1}=B^{-1}A^{-1}$$

$$\left(A^{T}\right)^{-1}=\left(A^{-1}\right)^{T}$$

vi) Finding A and A^{-1} from adj(A)

$$A^{-1}=\pm\frac{1}{\sqrt{|adjA|}}adj(A)$$

$$A=\pm\frac{1}{\sqrt{|adjA|}}adj(adjA)$$

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1) If $A=\begin{pmatrix}5&3\\-1&-2\end{pmatrix}$, Show that $A^{2}-3A-7I_{2}=O_{2}$,

Hence Find A^{-1}.

Ans:

$$A^{2}=\begin{pmatrix}5&3\\-1&-2\end{pmatrix}\begin{pmatrix}5&3\\-1&-2\end{pmatrix}=\begin{pmatrix}22&9\\-3&1\end{pmatrix}$$

$$A^{2}-3A-7I_{2}=\begin{pmatrix}22&9\\-3&1\end{pmatrix}+\begin{pmatrix}-15&-9\\3&6\end{pmatrix}+\begin{pmatrix}-7&0\\0&-7\end{pmatrix}$$

$$=\begin{pmatrix}0&0\\0&0\end{pmatrix}$$

$$A^{2}-3A-7I_{2}=O_{2}$$

Pre-multiply by A^{-1}

$$A-3I-7A^{-1}=O$$

$$A^{-1}=\frac{1}{7}(A-3I)=\frac{1}{7}\begin{pmatrix}2&3\\-1&-5\end{pmatrix}$$

2) Verify the property $\left(A^{T}\right)^{-1}=\left(A^{-1}\right)^{T}$ with $\begin{pmatrix}2&9\\1&7\end{pmatrix}$

Ans: $A^{T}=\begin{pmatrix}2&1\\9&7\end{pmatrix}$ and $\left|A^{T}\right|=5$

$$\left(A^{T}\right)^{-1}=\frac{1}{5}\begin{pmatrix}7&-1\\-9&2\end{pmatrix}\to(1)$$

$$A^{-1}=\frac{1}{5}\begin{pmatrix}7&-9\\-1&2\end{pmatrix}$$

$$\Rightarrow\left(A^{-1}\right)^{T}=\frac{1}{5}\begin{pmatrix}7&-1\\-9&2\end{pmatrix}\to(2)$$

$\therefore$ from (1) and (2) $\left(A^{T}\right)^{-1}=\left(A^{-1}\right)^{T}$

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3) If $A=\frac{1}{7}\begin{bmatrix}6&-3&a\\b&-2&6\\2&c&3\end{bmatrix}$ is orthogonal find a , b and c

and hence find A^{-1}.

Ans:

A is Orthogonal $\Rightarrow AA^{T}=I$

$$\begin{bmatrix}a^{2}+45&6a+6b+6&3a-3c+12\\6a+6b+6&b^{2}+40&2b-2c+18\\3a-3c+12&2b-2c+18&c^{2}+13\end{bmatrix}=\begin{bmatrix}49&0&0\\0&49&0\\0&0&49\end{bmatrix}$$

Solving

$a+b=-1$

$a-c=-4$ we get a=2 ; b=-3 ; c=6

$b-c=-9$

$$\therefore A^{-1}=A^{T}=\frac{1}{7}\begin{bmatrix}6&-3&2\\-3&-2&6\\2&6&3\end{bmatrix}$$

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4) If $adj(A)=\begin{bmatrix}-1&2&2\\1&1&2\\2&2&1\end{bmatrix}$, Find A^{-1}.

Ans:

$$|adjA|=9\quad\sqrt{|adjA|}=3$$

$$A^{-1}=\pm\frac{1}{\sqrt{|adjA|}}adj(A)$$

$$=\pm\frac{1}{3}\begin{bmatrix}-1 & 2 & 2\\ 1 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$$

5) **If** $adj(A)=\begin{bmatrix}2 & -4 & 2\\ -3 & 12 & -7\\ -2 & 0 & 2\end{bmatrix}$ **, find A**

Ans:

$$|adjA|=16\Rightarrow\sqrt{|adjA|}=4$$

$$A=\pm\frac{1}{\sqrt{|adjA|}}adj(adjA)$$

$$adj(adjA)=\begin{bmatrix}24 & 8 & 4\\ 20 & 8 & 8\\ 24 & 8 & 12\end{bmatrix}$$

$$A=\pm\frac{1}{4}\begin{bmatrix}24 & 8 & 4\\ 20 & 8 & 8\\ 24 & 8 & 12\end{bmatrix}=\pm\begin{bmatrix}6 & 2 & 1\\ 5 & 2 & 2\\ 6 & 2 & 3\end{bmatrix}$$

6) **If** $A=\begin{pmatrix}1 & \tan x\\ -\tan x & 1\end{pmatrix}$ **, show that**

$$A^T A^{-1}=\begin{bmatrix}\cos 2x & -\sin 2x\\ \sin 2x & \cos 2x\end{bmatrix}$$

Ans:

$$A^T=\begin{pmatrix}1 & -\tan x\\ \tan x & 1\end{pmatrix}$$

$$|A|=1+\tan^2 x=\sec^2 x$$

$$A^{-1}=\frac{1}{|A|}adjA=\frac{1}{\sec^2 x}\begin{pmatrix}1 & -\tan x\\ \tan x & 1\end{pmatrix}$$

$$=\cos^2 x\begin{pmatrix}1 & -\tan x\\ \tan x & 1\end{pmatrix}=\begin{pmatrix}\cos^2 x & -\sin x\cos x\\ \sin x\cos x & \cos^2 x\end{pmatrix}$$

$$A^T A^{-1}=\begin{bmatrix}1 & -\tan x\\ \tan x & 1\end{bmatrix}\begin{bmatrix}\cos^2 x & -\sin x\cos x\\ \sin x\cos x & \cos^2 x\end{bmatrix}$$

$$=\begin{pmatrix}\cos 2x & -\sin 2x\\ \sin 2x & \cos 2x\end{pmatrix}$$

7) **Decrypt the received encoded message** $[2\ \ -3],[20\ \ 4]$ **with the encryption matrix** $\begin{pmatrix}-1 & -1\\ 2 & 1\end{pmatrix}$ **and the decryption matrix as its inverse where the system of code are described by the numbers 1 - 26 to the letters A - Z respectively, and the number 0 to a blank space.**

Ans: Encoding Matrix

$$A=\begin{pmatrix}-1 & -1\\ 2 & 1\end{pmatrix},\Rightarrow A^{-1}=\begin{pmatrix}1 & 1\\ -2 & -1\end{pmatrix}$$

Coded row matrix	Decoding matrix	Decoded row matrix
$[2\ \ -3]$	$\begin{pmatrix}1 & 1\\ -2 & -1\end{pmatrix}$	$[8\ \ 5]$
$[20\ \ 4]$	$\begin{pmatrix}1 & 1\\ -2 & -1\end{pmatrix}$	$[12\ \ 16]$

In Alphabetical order $[8\ \ 5][12\ \ 16]\Rightarrow HELP$

Unit II

- Row-Echelon form
- Rank - Minor Method
- Rank - Row Echelon Method
- Gauss Jordan Method:
  For any Non Singular Matrix A
  $$[A]\rightarrow[I_n]$$
- Finding Inverse - Gauss Jordan Method
  $$[A/I_n]\rightarrow[I_n/A^{-1}]\ ;\ |A|\neq 0$$

8) **Find the rank of the matrix** $\begin{bmatrix}0 & 1 & 2 & 1\\ 0 & 2 & 4 & 3\\ 8 & 1 & 0 & 2\end{bmatrix}$ **by minor method.**

Ans:

$$A=\begin{bmatrix}0 & 1 & 2 & 1\\ 0 & 2 & 4 & 3\\ 8 & 1 & 0 & 2\end{bmatrix}\Rightarrow\rho(A)\le 3$$

$$\begin{vmatrix}0 & 1 & 2\\ 0 & 2 & 4\\ 8 & 1 & 0\end{vmatrix}=0\ But\ \begin{vmatrix}1 & 2 & 1\\ 2 & 4 & 3\\ 1 & 0 & 2\end{vmatrix}=2\neq 0$$

$$\therefore\rho(A)=3$$

9) **Find the rank of the matrix** $\begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$ **by row- echelon method.**

Ans:

$$A=\begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$$

$$\sim\begin{bmatrix} 1 & -2 & -3 & 2 \\ 2 & -5 & 1 & 4 \\ 3 & -8 & 5 & 2 \end{bmatrix} \quad R_1 \leftrightarrow R_3$$

$$\sim\begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & -1 & 7 & 0 \\ 0 & -2 & 14 & -4 \end{bmatrix} \quad -2R_1+R_2 \to R_2$$

$$\sim\begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & -1 & 7 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix} \quad -2R_2+R_3 \to R_3$$

$$\therefore \rho(A)=3$$

10) **Find the inverse of** $\begin{pmatrix} 2 & -1 \\ 5 & -2 \end{pmatrix}$ **by Gauss Jordan Method.**

Ans:

$$|A|=\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}=-4+5=1\neq 0$$

$$[A/I]=\left[\begin{array}{cc|cc} 2 & -1 & 1 & 0 \\ 5 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_1\to R_1}\left[\begin{array}{cc|cc} 1 & \frac{-1}{2} & \frac{1}{2} & 0 \\ 5 & -2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{-5R_1+R_2\to R_2}\left[\begin{array}{cc|cc} 1 & \frac{-1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{-5}{2} & 1 \end{array}\right]$$

$$\xrightarrow{R_1+R_2\to R_1}\left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & \frac{1}{2} & \frac{-5}{2} & 1 \end{array}\right]$$

$$\xrightarrow{2R_2\to R_2}\left[\begin{array}{cc|cc} 1 & 0 & -2 & 1 \\ 0 & 1 & -5 & 2 \end{array}\right]\to\left[I/A^{-1}\right]$$

$$\therefore A^{-1}=\begin{pmatrix} -2 & 1 \\ -5 & 2 \end{pmatrix}$$

11) **Find the inverse of the matrix** $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$ **by Gauss Jordan method.**

Ans:

$$[A/I]=\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$$

$$\sim\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right] \begin{pmatrix} -2R_1+R_2\to R_2 \\ -R_1+R_3\to R_3 \end{pmatrix}$$

$$\sim\left[\begin{array}{ccc|ccc} 1 & 0 & 8 & 0 & 0 & 1 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right] \begin{pmatrix} R_1+R_2\to R_2 \\ 2R_2+R_3\to R_3 \end{pmatrix}$$

$$\sim\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right] \begin{pmatrix} R_1+8R_3\to R_1 \\ -1\times R_3\to R_3 \end{pmatrix}$$

$$\sim\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right] \quad R_2+3R_3\to R_2$$

$$\therefore A^{-1}=\begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

Unit:III

- **Solving system of linear equations by Matrix Inversion Method**

  $A\times X=B$

- $|A|\neq 0$ **(only Unique solution)**

  $\therefore X=A^{-1}B$

- **Application problems**

12) **Solve the system by Matric Inversion method** $2x_1+3x_2+3x_3=5$; $x_1-2x_2+x_3=-4$ **and** $3x_1-x_2-2x_3=3$

Ans:

$$\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix} \Rightarrow AX = B$$

$$|A| = 40 \neq 0$$

$$adj(A) = \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adjA = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

13) A man is appointed in a job with a monthly salary of certain amount and a fixed amount of annual increment. If his salary was Rs.19800 per month at the end of the first month after 3 years of service and Rs.23400 per month at the end of the first month after 9 years of service, find his starting salary and his annual increment. (Use matrix inversion method)

Ans:

$$x + 3y = 19800$$
$$x + 9y = 23400$$

$$\begin{bmatrix} 1 & 3 \\ 1 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19800 \\ 23400 \end{bmatrix} \Rightarrow AX = B$$

$$|A| = 6 \Rightarrow A^{-1} = \frac{1}{6}\begin{bmatrix} 9 & -3 \\ -1 & 1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 9 & -3 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 19800 \\ 23400 \end{bmatrix} = \begin{bmatrix} 18000 \\ 600 \end{bmatrix}$$

14) 4 men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

Ans:

Work finished by a man in one day = $\frac{1}{x}$

Work finished by a woman in one day = $\frac{1}{y}$

Given: $\frac{4}{x} + \frac{4}{y} = \frac{1}{3}$; $\frac{2}{x} + \frac{5}{y} = \frac{1}{4}$

Take $\frac{1}{x}$ = a and $\frac{1}{y}$ = b

$$4a + 4b = \frac{1}{3} \Rightarrow 12a + 12b = 1$$

$$2a + 5b = \frac{1}{4} \Rightarrow 8a + 20b = 1$$

$$\begin{bmatrix} 12 & 12 \\ 8 & 20 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow AX = B$$

$$|A| = 144 \Rightarrow A^{-1} = \frac{1}{144}\begin{bmatrix} 20 & -12 \\ -8 & 12 \end{bmatrix}$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{144}\begin{bmatrix} 20 & -12 \\ -8 & 12 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1/18 \\ 1/36 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 18 \\ 36 \end{bmatrix}$$

Unit: IV

- **Solving system of linear equations by Cramer's Rule**

  Coefficient determinant $\Delta \neq 0$

  we can apply Cramer's rule & only Unique solution exist.

$$x = \frac{\Delta_x}{\Delta}$$

$$y = \frac{\Delta_y}{\Delta}$$

- **Application problems using Cramer's rule**

15) Solve by Cramer's rule:

$\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0$; $\frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0$ **and**

$\frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$

Ans:

Put $\frac{1}{x}=a$; $\frac{1}{y}=b$; $\frac{1}{z}=c$

$$3a-4b-2c=1$$
$$a+2b+c=2$$
$$2a-5b-4c=-1$$

$$\Delta=\begin{bmatrix}3 & -4 & -2\\ 1 & 2 & 1\\ 2 & -5 & -4\end{bmatrix}=-15\neq 0$$

$$\Delta_a=\begin{bmatrix}1 & -4 & -2\\ 2 & 2 & 1\\ -1 & -5 & -4\end{bmatrix}=-15$$

$$\Delta_b=\begin{bmatrix}3 & 1 & -2\\ 1 & 2 & 1\\ 2 & -1 & -4\end{bmatrix}=-5$$

$$\Delta_c=\begin{bmatrix}3 & -4 & 1\\ 1 & 2 & 2\\ 2 & -5 & -1\end{bmatrix}=-5$$

by Cramer's rule

$a=\frac{\Delta_a}{\Delta}$; $b=\frac{\Delta_b}{\Delta}$; $c=\frac{\Delta_c}{\Delta}$

$a=1$; $b=\frac{1}{3}$; $c=\frac{1}{3}$

$x=1$; $y=3$; $z=3$

16) In a competitive examination, one mark is awarded for every correct answer while 1/4 mark is deducted for every wrong answer. A student answered 100 questions and get 80 marks. How many questions did he answer correctly? (Use Cramer's rule)

Ans:

Given:

$$x+y=100$$

$$x-\frac{1}{4}y=80\Rightarrow 4x-y=320$$

$$\Delta=\begin{vmatrix}1 & 1\\ 4 & -1\end{vmatrix}=-5$$

$$\Delta_x=\begin{vmatrix}100 & 1\\ 320 & -1\end{vmatrix}=-420$$

$$\Delta_y=\begin{vmatrix}1 & 100\\ 4 & 320\end{vmatrix}=-80$$

$$x=\frac{\Delta_x}{\Delta}=84\ \&\ y=\frac{\Delta_y}{\Delta}=16$$

17) A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 liters of a 40% acid solution? (Use Cramer's rule)

Ans:

Given: $x+y=10$

$$\frac{50}{100}x+\frac{25}{100}y=\frac{40}{100}\times 10\Rightarrow 50x+25y=400$$

$$\Delta=\begin{vmatrix}1 & 1\\ 50 & 25\end{vmatrix}=-25$$

$$\Delta_x=\begin{vmatrix}10 & 1\\ 400 & 25\end{vmatrix}=-150$$

$$\Delta_y=\begin{vmatrix}1 & 10\\ 50 & 400\end{vmatrix}=-100$$

$$x=\frac{\Delta_x}{\Delta}=6\ \&\ y=\frac{\Delta_y}{\Delta}=4$$

18) A fish tank can be filled in 10 minutes using both pumps A and B simultaneously. However, pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes. How long would it take each pump to fill the tank by itself? (Use Cramer's rule)

Ans:

Amount of water filled by pump A in 1m = $\frac{1}{x}$

Amount of water filled by pump B in 1m = $\frac{1}{y}$

Given: $\frac{1}{x}+\frac{1}{y}=\frac{1}{10}\ \&\ \frac{1}{x}-\frac{1}{y}=\frac{1}{30}$

Take $\frac{1}{x}=a\ \&\ \frac{1}{y}=b$

$$10a+10b=1$$
$$30a-30b=1$$
$$\Delta=\begin{vmatrix}10 & 10\\ 30 & -30\end{vmatrix}=-600$$
$$\Delta_x=\begin{vmatrix}1 & 10\\ 1 & -30\end{vmatrix}=-40$$
$$\Delta_y=\begin{vmatrix}10 & 1\\ 30 & 1\end{vmatrix}=-20$$
$$a=\frac{\Delta_a}{\Delta} \quad \& \quad b=\frac{\Delta_b}{\Delta}$$
$$\frac{1}{x}=\frac{1}{15} \quad \& \quad \frac{1}{y}=\frac{1}{30}$$
$$x=15 \quad \& \quad y=30$$

Unit:V

- **Gaussian Elimination method:**
- **Find Augmented matrix and write it in Row Echelon form**

$$[A/B]=\left[\begin{array}{ccc|c} a_1 & b_1 & c_1 & d_1\\ a_2 & b_2 & c_2 & d_2\\ a_3 & b_3 & c_3 & d_3\end{array}\right]\sim\left[\begin{array}{ccc|c} a_1 & b_1 & c_1 & d_1\\ 0 & b_2 & c_2 & d_2\\ 0 & 0 & c_3 & d_3\end{array}\right]$$

19) Solve the following systems of linear equations by Gaussian elimination method.

$2x-2y+3z=2$; $x+2y-z=3$; $3x-y+2z=1$

Ans:

$$[A/B]=\left[\begin{array}{ccc|c} 2 & -2 & 3 & 2\\ 1 & 2 & -1 & 3\\ 3 & -1 & 2 & 1\end{array}\right]$$
$$\sim\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3\\ 0 & -6 & 5 & -4\\ 0 & 0 & 1 & 4\end{array}\right]$$

Converting into system of equations

$$x+2y-z=3$$
$$-6y+5z=-4$$
$$z=4$$

Solving we get

$$(x,y,z)=(-1,4,4)$$

20) If ax^2+bx+c is divided by x+3, x-5 and x-1 the remainders are 21, 61 and 9 respectively. Find a, b and c (Use Gaussian Elimination method)

Ans:

$$p(x)=ax^2+bx+c$$
$$p(-3)=21\Rightarrow 9a-3b+c=21$$
$$p(5)=61 \Rightarrow 25a+5b+c=61$$
$$p(1)=9 \Rightarrow a+b+c=9$$
$$[A/B]=\left[\begin{array}{ccc|c} 9 & -3 & 1 & 21\\ 25 & 5 & 1 & 61\\ 1 & 1 & 1 & 9\end{array}\right]$$
$$\sim\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9\\ 0 & 5 & 6 & 41\\ 0 & 0 & 1 & 6\end{array}\right]$$
$$a+b+c=9$$
$$5b+6c=41$$
$$c=6$$
$$\therefore (a,b,c)=(2,1,6)$$

21) A boy through the points (-6,8), (-2,-12), (3,8). He wants to meet his friend at P(7,60). Will he meet his friend? (Use Gaussian Elimination method)

Ans: Given: $y=ax^2+bx+c$

$$(x,y)=(-6,8) \Rightarrow 36a-6b+c=8$$
$$(x,y)=(-2,-12)\Rightarrow 4a-2b+c=-12$$
$$(x,y)=(3,8) \Rightarrow 9a+3b+c=8$$
$$[A/B]=\left[\begin{array}{ccc|c} 36 & -6 & 1 & 8\\ 4 & -2 & 1 & -12\\ 9 & 3 & 1 & 8\end{array}\right]$$
$$[A/B]=\left[\begin{array}{ccc|c} 36 & -6 & 1 & 8\\ 0 & -3 & 2 & -29\\ 0 & 0 & 1 & -10\end{array}\right]$$
$$36a-6b+c=8$$
$$-3b+2c=-29$$
$$c=-10$$

Solving we get

$(a,b,c)=(1,3,-10) \Rightarrow y=x^2+3x-10$

$x=7 \Rightarrow y=60$

The point (7,60) satisfies the equation, Hence the boy will meet his friend at P(7,60)

Unit: VI

- **Non-Homogenous equations-Testing Consistency**
- **Rouche's Capilli theorem**

  $\rho(A)=\rho[A/B] \Rightarrow$ **System is consistent**
- $\rho(A)=\rho[A/B]=3 \Rightarrow$ **Unique solution**
- $\left.\begin{array}{l}\rho(A)=\rho[A/B]=2\\ \rho(A)=\rho[A/B]=1\end{array}\right\}<3$ **Many Solutions**
- $\rho(A)\neq\rho[A/B] \Rightarrow$ **No solution**

22) Find the value of k for which the equations $kx-2y+z=1$; $x-2ky+z=-2$; $x-2y+kz=1$ **have i) no solution ii) Unique solution iii) infinitely many solution.**

Ans:

$$[A/B]=\begin{bmatrix} k & -2 & 1 & 1\\ 1 & -2k & 1 & -2\\ 1 & -2 & k & 1\end{bmatrix}$$

$$\sim\begin{bmatrix} 1 & -2 & k & 1\\ 0 & 2-2k & 1-k & -3\\ 0 & 2k-2 & 1-k^2 & 1-k\end{bmatrix}$$

$$\sim\begin{bmatrix} 1 & -2 & k & 1\\ 0 & 2-2k & 1-k & -3\\ 0 & 0 & (k+2)(1-k) & -(k+2)\end{bmatrix}$$

Case(i) When k=1

$\rho(A)\neq\rho[A/B]$ system has no solution

Case(ii) When $k+2\neq 0$

$\rho(A)=\rho[A/B]=3$ system has unique solution

Case(iii) When k+2=0

$\rho(A)=\rho[A/B]=2<3$ system has infinitely many solution.

Unit:VII

- **Homogeneous equations**

  -- Always Consistent
- **Trivial solution:** $|A|\neq 0$

  $\rho(A)=\rho[A/B]=3$

  Only trivial solution

  $\Rightarrow(x,y,z)=(0,0,0)$
- **Non-Trivial solution :** $|A|=0$

  $\left.\begin{array}{l}\rho(A)=\rho[A/B]=2\\ \rho(A)=\rho[A/B]=1\end{array}\right\}<3$

Infinitely many non-trivial solutions

23) By using Gaussian elimination method, balance the chemical reaction equation:

$C_2H_6+O_2\rightarrow H_2O+CO_2$

Ans:

$x_1C_2H_6+x_2O_2\rightarrow x_3H_2O+x_4CO_2$

$C\rightarrow 2x_1+0x_2+0x_3-x_4=0$

$H\rightarrow 3x_1+0x_2-x_3+0x_4=0$

$O\rightarrow 0x_1+2x_2-x_3-2x_4=0$

$$[A/O]=\begin{bmatrix} 2 & 0 & 0 & -1 & 0\\ 3 & 0 & -1 & 0 & 0\\ 0 & 2 & -1 & -2 & 0\end{bmatrix}$$

$$\sim\begin{bmatrix} 2 & 0 & 0 & -1 & 0\\ 0 & 2 & -1 & -2 & 0\\ 0 & 0 & -2 & 3 & 0\end{bmatrix}$$

$\rho(A)=\rho[A/O]=3<4$ system is consistent and has infinite number of solution.

$2x_1-x_4=0$

$2x_2-x_3-2x_4=0$

$-2x_3+3x_4=0$

$put\ x_4=t\Rightarrow(x_1,x_2,x_3,x_4)=\left(\frac{t}{2},\frac{7t}{4},\frac{3t}{2},t\right)$

$When\ t=4\Rightarrow(x_1,x_2,x_3,x_4)=(2,7,6,4)$

$\therefore 2C_2H_6+7O_2\rightarrow 6H_2O+4CO_2$

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